

Turbulent thermalization process in heavy-ion collisions at ultrarelativistic energies

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Thermalization process



Initial state:
Far from equilibrium

*Non-equilibrium
dynamics*

Final state:
Thermal equilibrium



How is thermal equilibrium achieved?

Motivation

Relativistic heavy-ion collision experiments at RHIC and LHC



Can we understand the complex dynamics in an *ab-initio* approach to heavy-ion collisions?

Heavy-ion collisions

Conjectured space-time evolution of a heavy-collision based on phenomenological models and experimental information

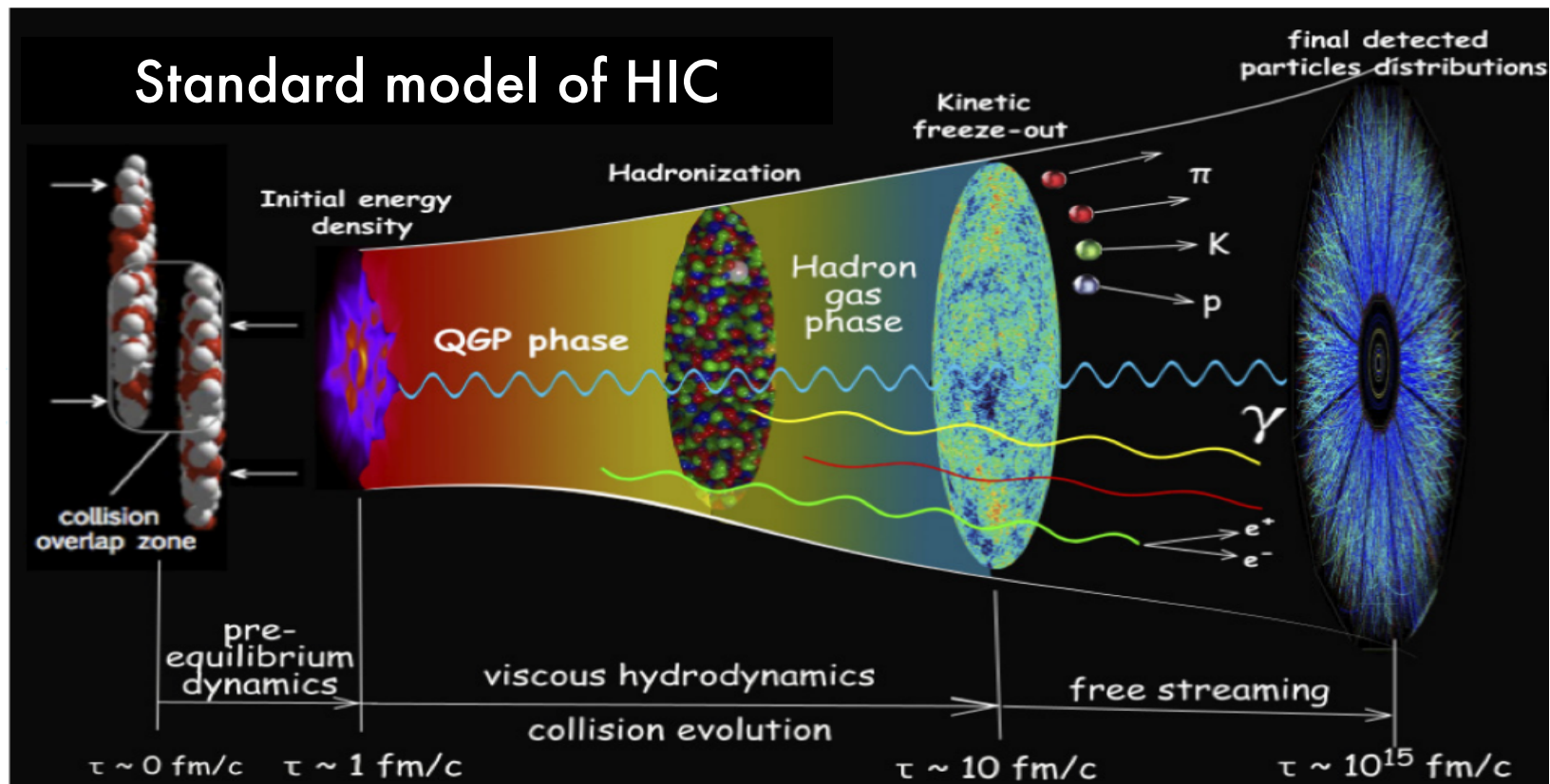
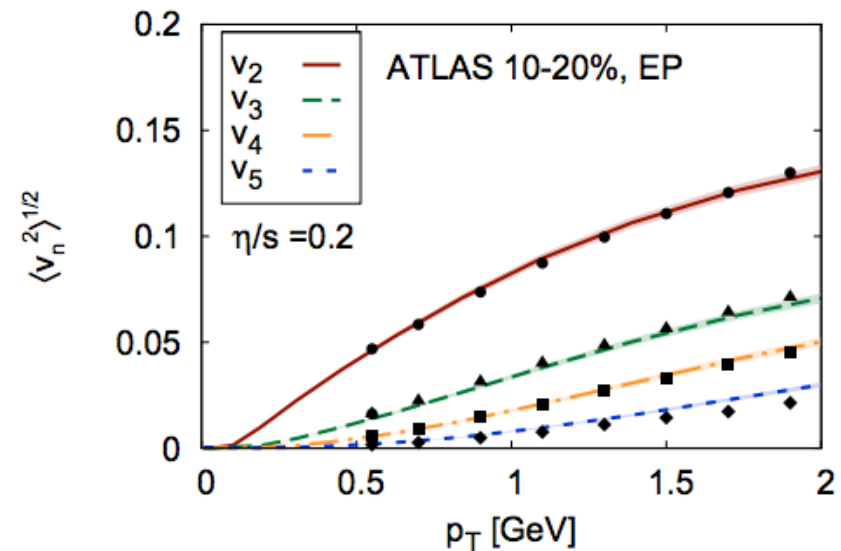
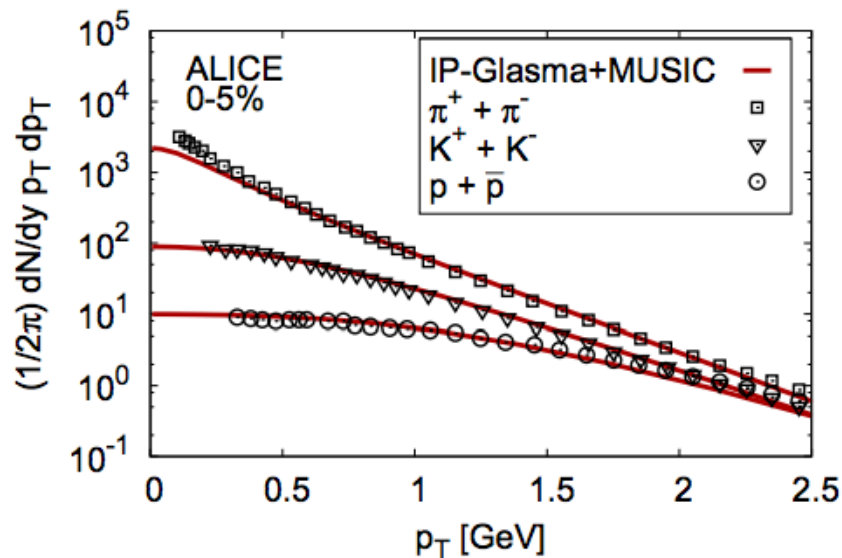


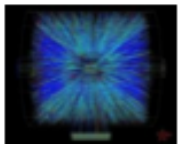
Fig. by P. Sorensen and C. Shen

Heavy-ion collisions

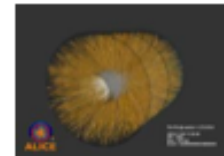
Hydrodynamic simulations versus experiment



Schenke et al. PRL 110 (2013) 012302



A large variety of data at RHIC and LHC can be explained based on this standard model



**When and to what extent is isotropization/thermalization achieved?
How does this happen?**

Heavy-ion collisions

Progress in a first-principle understanding from two limiting cases

Holographic thermalization:

a) strong coupling? *Heller, Janik, Witaszczyk; Chesler, Yaffe ...*

Sizeable anisotropy at transition to hydrodynamic regime

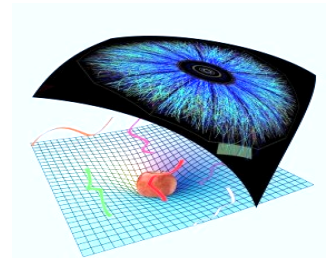


Fig. from strings.net.technion.ac.il

Turbulent thermalization:

b) weak coupling but highly occupied? *CGC: McLerran, Venugopalan ...*

Energy density of gluons with typical momentum Q_s (at time $\sim 1/Q_s$)

$$\epsilon \sim \frac{Q_s^4}{\alpha_s} \quad \text{i.e. 'occupation numbers'} \quad n(p \lesssim Q_s) \sim \frac{1}{\alpha_s}$$

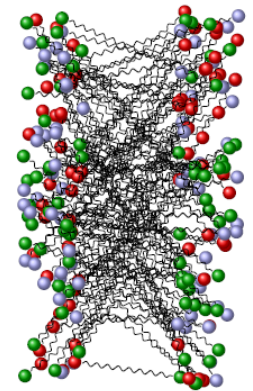


Fig. by T. Epelbaum

Non-equilibrium dynamics

Solve *Initial value problem* in QCD

- **Initial conditions:**

Inspired by **color glass condensate** (CGC) description of heavy ion collisions ($n(p) \sim 1/\alpha$)

- **Non-equilibrium dynamics:**

- Classical-statistical lattice simulations (numerical studies) ($n(p) \gg 1$)

- Kinetic theory ($n(p) < 1/\alpha$) (analytic discussion)

Far from
equilibrium

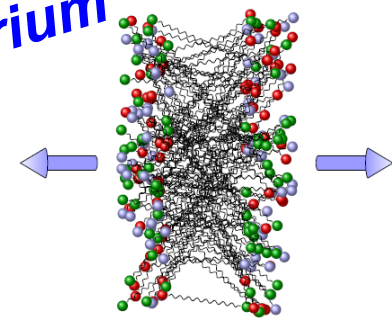


Fig. by T. Epelbaum

First principle
Intuitive picture

Non-equilibrium dynamics



Initial state:
Far from equilibrium



*Non-equilibrium
dynamics*



Final state:
Thermal equilibrium



How is thermal equilibrium achieved?

Thermalization process

Non-equilibrium phenomena may be shared by a large class of strongly correlated many-body systems

I) Thermalization in scalar field theory – Cosmology

(Micha, Tkachev PRD 70 (2004) 043538)

(Berges, Boguslavski, SS, Venugopalan arXiv:1312.5216)

II) Thermalization in Yang-Mills theory in Minkowski space

(Berges, SS, Sexty PRD 86 (2012) 074006; SS PRD 86 (2012) 065008)

III) Thermalization in heavy-ion collisions at ultra-relativistic energies – weak coupling, large nuclei

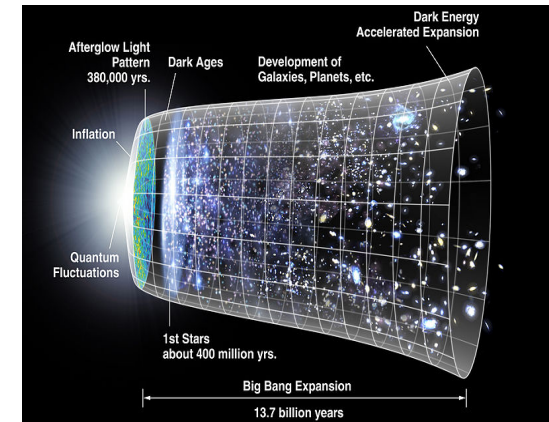
(Berges, Boguslavski, SS, Venugopalan arXiv:1303.5650, arXiv:1311.3005)

Thermalization process - Cosmology

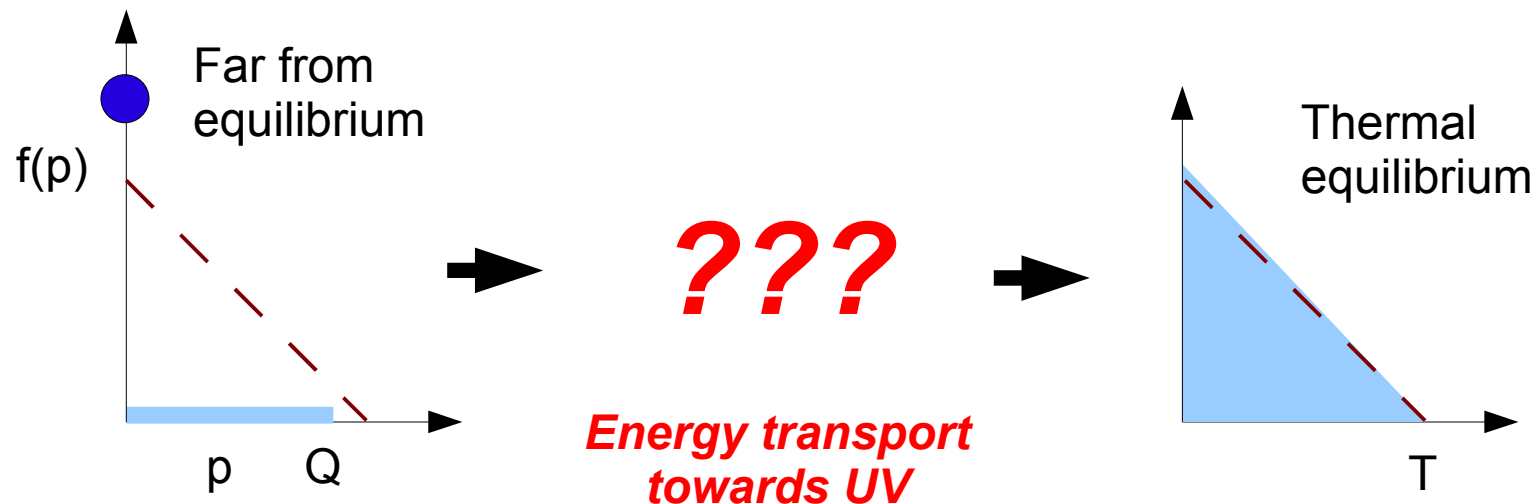
Model for thermalization of the early universe:

Scalar field theory ($\lambda\Phi^4$); Small coupling $\lambda = 10^{-8}$.

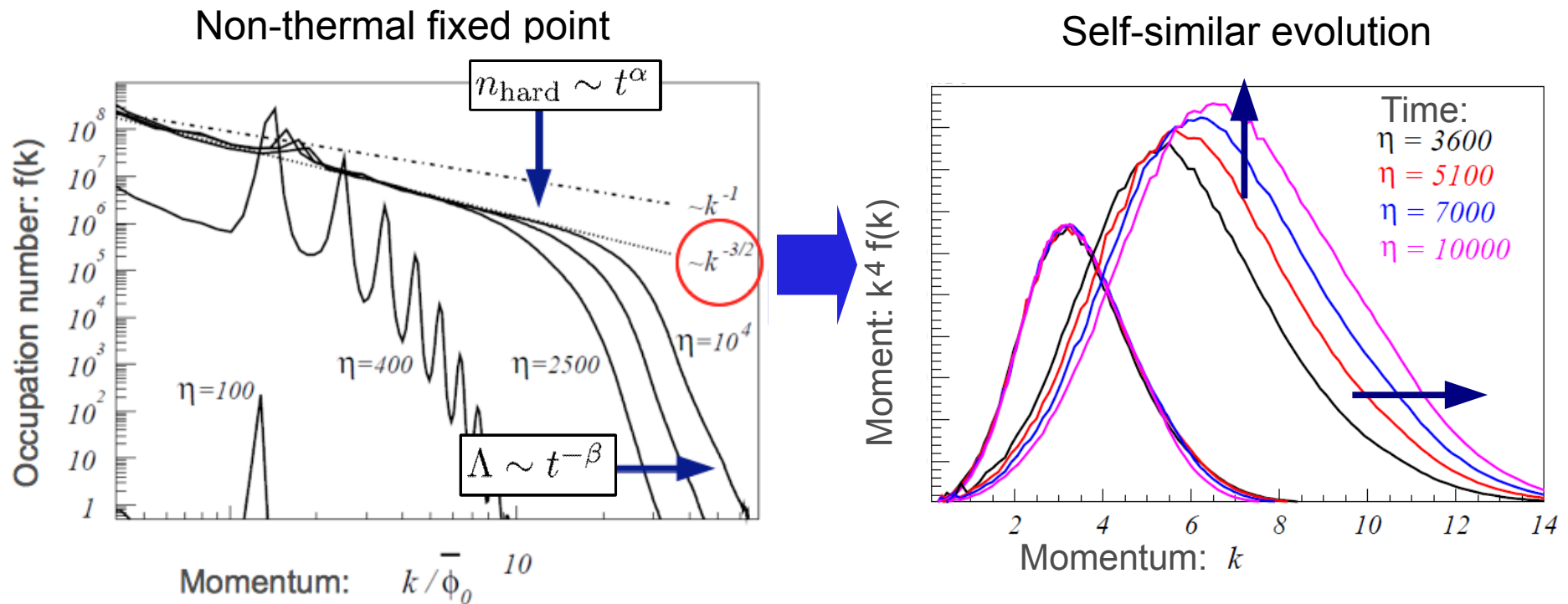
$$S[\varphi] = \int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{\lambda}{24} \varphi^4 \right)$$



At the end of inflation: Background field $\Phi_0 \sim 1/\sqrt{\lambda}$ + vacuum fluctuations



Thermalization process - Cosmology

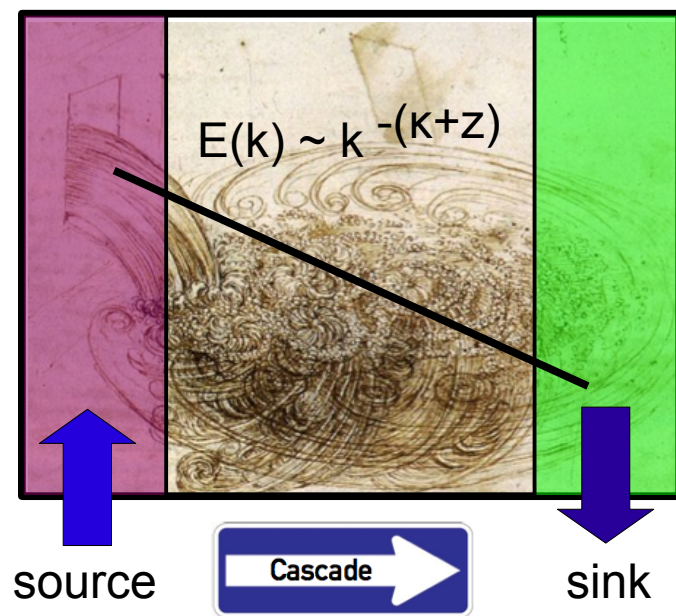


- The evolution becomes **self-similar** $f(p, t) = t^\alpha f_S(t^\beta p)$
- The thermalization process is described by a **quasi-stationary evolution** with **scaling exponents** Dynamic: $\alpha=-4/5$ $\beta=-1/5$ Spectral: $\kappa=-3/2$

(Micha, Tkachev PRD 70 (2004) 043538)

Manifestation of turbulence

“Driven” Turbulence – Kolmogorov wave turbulence



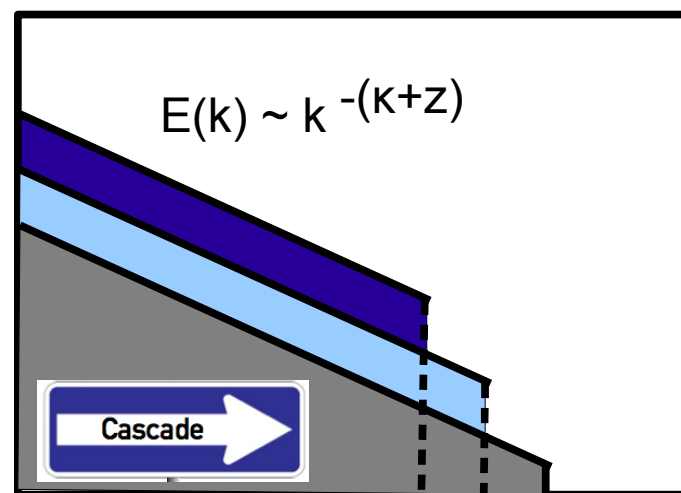
- **Stationary scaling solution** associated to scale invariant energy flux

Uriel Frisch, *“Turbulence. The Legacy of A. N. Kolmogorov.”*

Zakharov, V. E.; L'vov, V. S.; Falkovich, G, *“Kolmogorov spectra of turbulence 1. Wave turbulence.”*

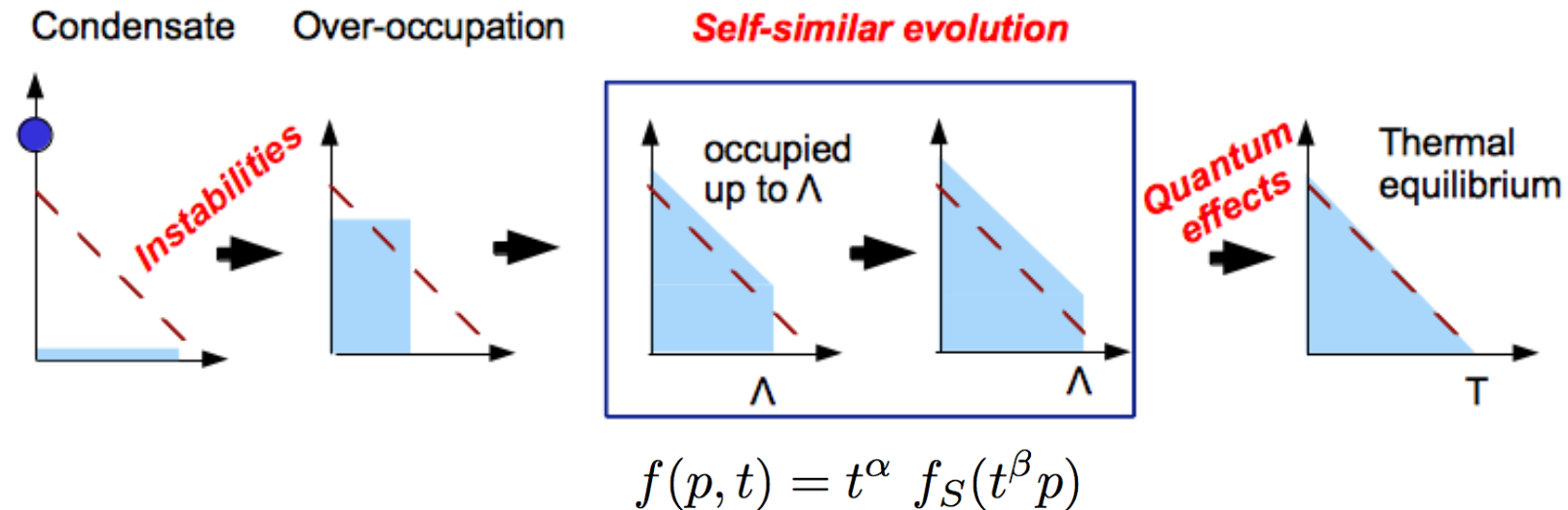
vs.

“Free” Turbulence – Turbulent Thermalization



- **quasi-stationary solution** with universal **non-thermal spectral exponents**
- **Self-similar evolution** with universal **dynamical scaling exponents**

Turbulent thermalization



Kinetic theory:

- Search for **self-similar scaling solutions** of the Boltzmann equation $\partial_t f(p, t) = C[f](p, t)$

- Fixed point equation + Scaling relation

$$\alpha f_S(p) + \beta \partial_p f_S(p) = C[f_S](p) \quad \alpha - 1 = \mu(\alpha, \beta)$$




(Cosmology: Micha, Tkachev PRD 70 (2004) 043538)

Turbulent thermalization

- The **scaling exponents** are **universal** numbers, determined by

$$\alpha - 1 = \mu(\alpha, \beta) \quad + \text{ conservation laws}$$

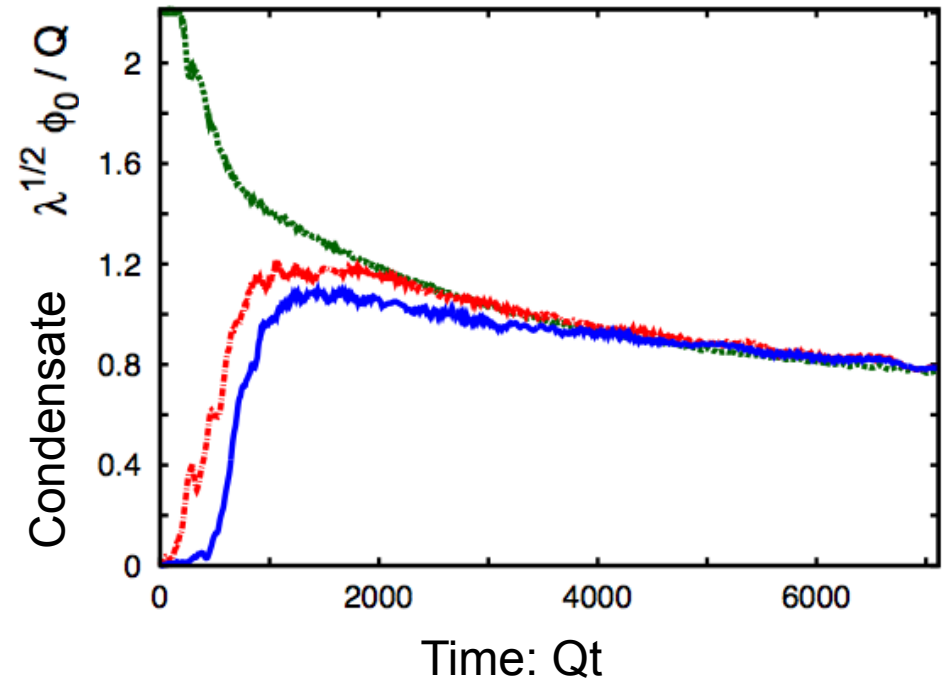
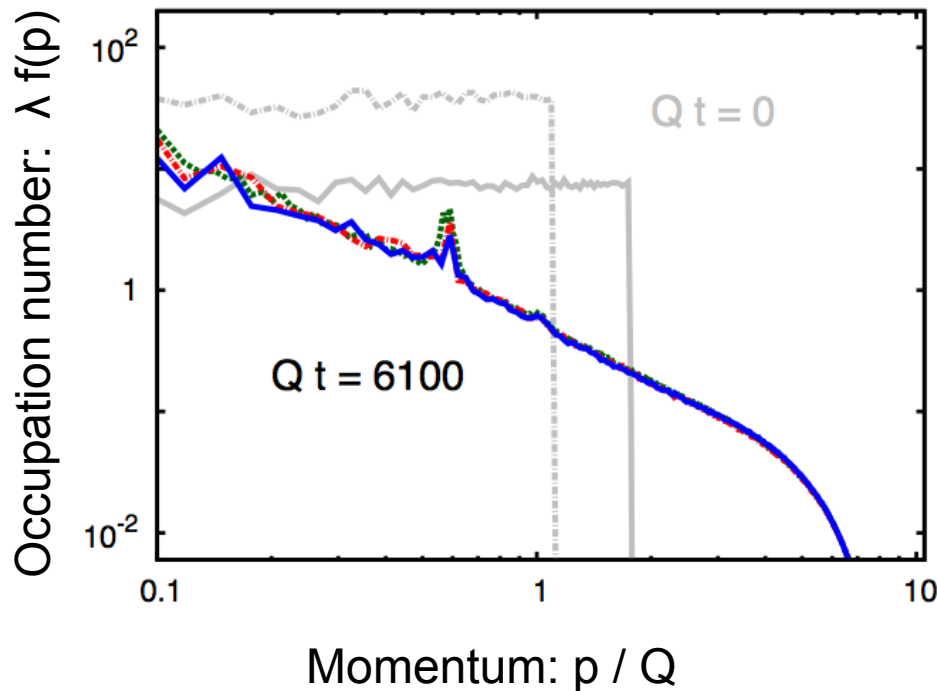
*Scaling properties of the
collision integral*

Interaction	Spectral Shape (Exponent κ)	Λ evolution (Exponent α)	Occupancy evolution (Exponent β)
 2<->1+soft	3/2	-1/5	-4/5
 2<->2	4/3	-1/7	-4/7
 2<->3 (gauge theory)	??	-1/7	-4/7

- Scalar theory:** turbulent cascade is driven by **2<->(1+soft)** interaction

(Cosmology: Micha, Tkachev PRD 70 (2004) 043538)

Independence of Initial conditions

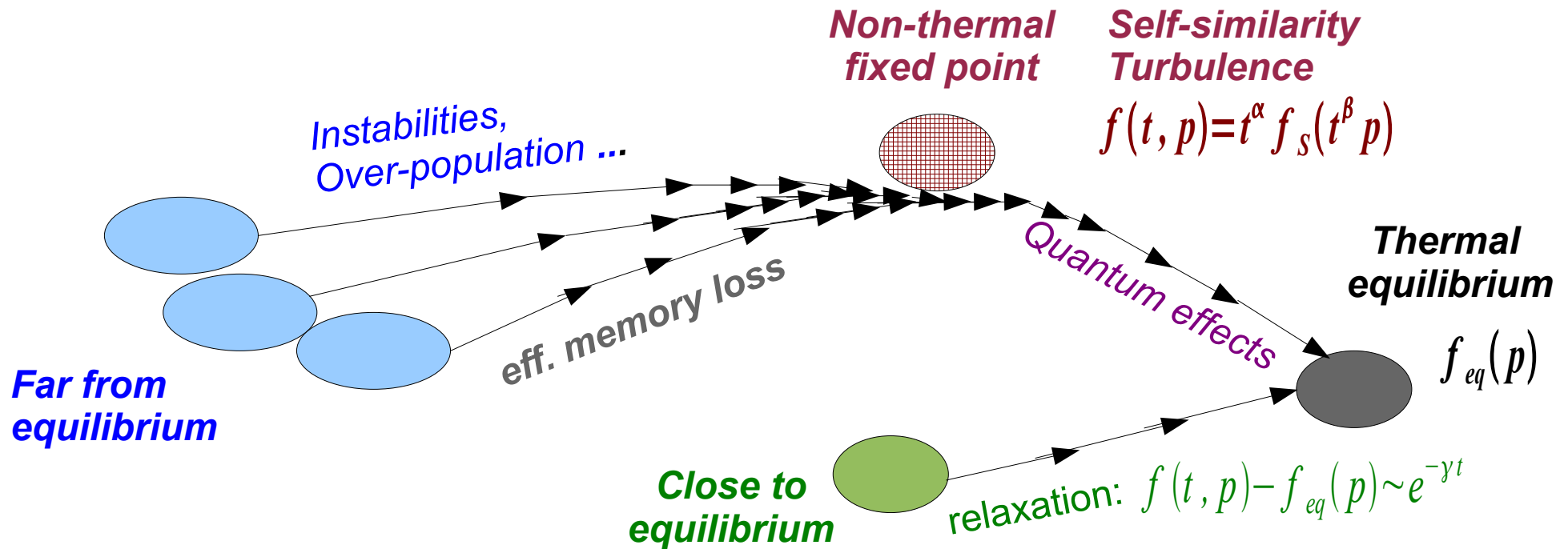


- The turbulent scaling behavior is really a property of the thermalization process – ***independent of the underlying initial conditions***
- An ***effective memory loss*** occurs ***already at the early stages*** of the thermalization process

(Berges, Boguslavski, SS, Venugopalan [arXiv:1312.5216](https://arxiv.org/abs/1312.5216))

Turbulent thermalization

Thermalization for a system far from equilibrium proceeds as a **self-similar evolution** associated to the presence of a **non-thermal fixed point**



*How does this picture apply to non-Abelian gauge theories?
Does it hold for relativistic heavy-ion collisions?*

Non-abelian plasma – “Static Box”

- Consider homegenous and *isotropic* systems which are initially *highly occupied* and initially characterized by a single momentum scale Q



How does thermalization proceed? Turbulence? What are the relevant kinetic processes?

(c.f. Kurkela, Moore *JHEP* 1112 (2011) 044; Blaizot et al. *Nucl.Phys. A*873 (2012) 68-80)

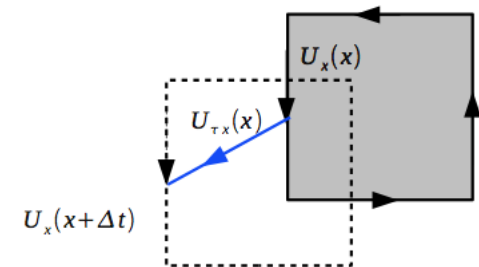
Classical-statistical lattice simulations

- **Initial conditions** chosen to mimic quasi-particle picture

$$A_{\mu}^a(t_0, \mathbf{x}) = \sum_{\lambda=1,2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sqrt{f(\mathbf{k}, t_0)} \left[c_{\lambda,a}^{\mathbf{k}} \xi_{\mu}^{(\lambda)\mathbf{k}+}(t_0) e^{i\mathbf{k}\mathbf{x}} + c.c. \right] ,$$

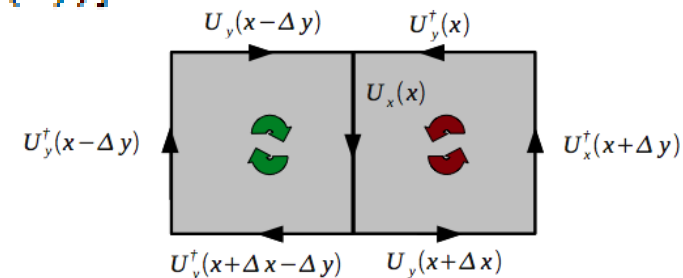
- Solve **equations of motion** based on Kogut-Susskind lattice Hamiltonian for SU(2) gauge group

$$U_i(x + \hat{t}) = \exp \left[i \frac{a_t}{a} \tilde{E}_a^i(x) \Gamma^a \right] U_i(x) ,$$



$$\tilde{E}_a^i(x) - \tilde{E}_a^i(x - \hat{t}) = 2 \frac{a_t}{a} \sum_{j \neq i} \text{tr} [i \Gamma^a (V_{ij}(x) + W_{ij}(x))]$$

(see e.g. Berges, Boguslavski, SS Venugopalan arXiv:1311.3005)



Occupation number

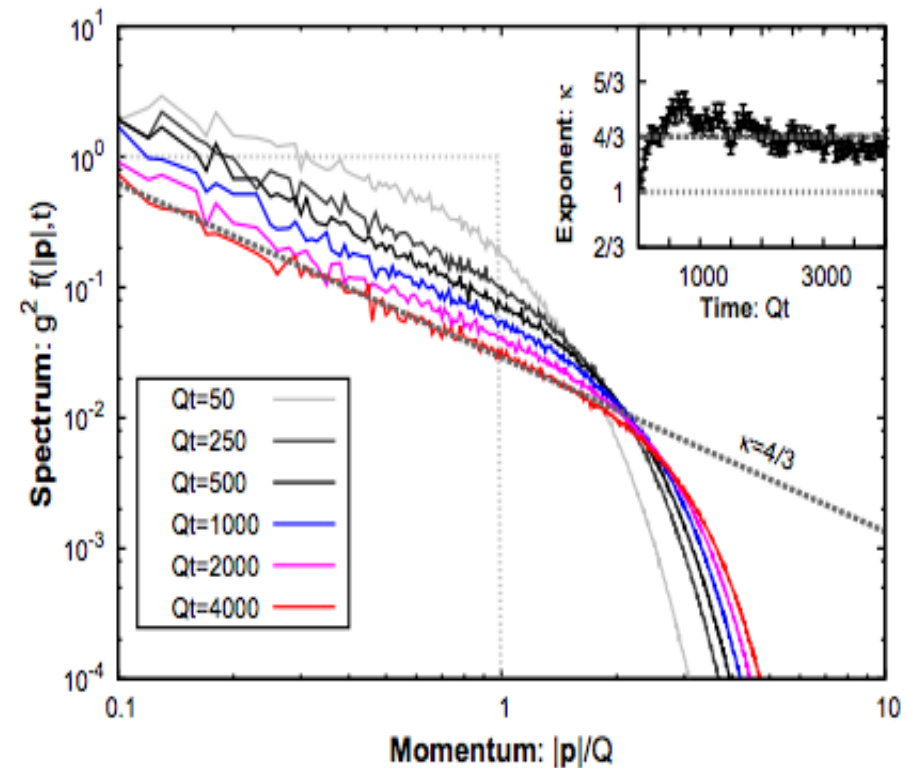
- Non-perturbative and non-equilibrium calculation -- occupation number is a ***gauge dependent*** quantity
- Chose ***temporal axial + Coulomb type gauge*** to fix the gauge freedom

$$A_t = 0 \quad \nabla \cdot \mathbf{A} \Big|_t = 0$$

- Define occupation number from ***equal time correlation functions***

$$f(\mathbf{p}, t) = \frac{1}{N_g (Na)^3} \sum_{a=1}^{N_c^2-1} \sum_{\lambda=1,2} \left\langle \left| \left(\xi_{\mu}^{(\lambda)\mathbf{p}+}(t) \right)^* \overleftrightarrow{\partial}_t A_a^{\mu}(t, \mathbf{p}) \right|^2 \right\rangle_{\text{Coul. Gauge}},$$

(Berges, Scheffler, Sexty *PLB* 681 (2009) 362-366; Berges, SS, Sexty *arXiv:1203.4646* (2012); SS *arXiv:1207.1450* (2012); Kurkela, Moore *arXiv:1207.1663* (2012))



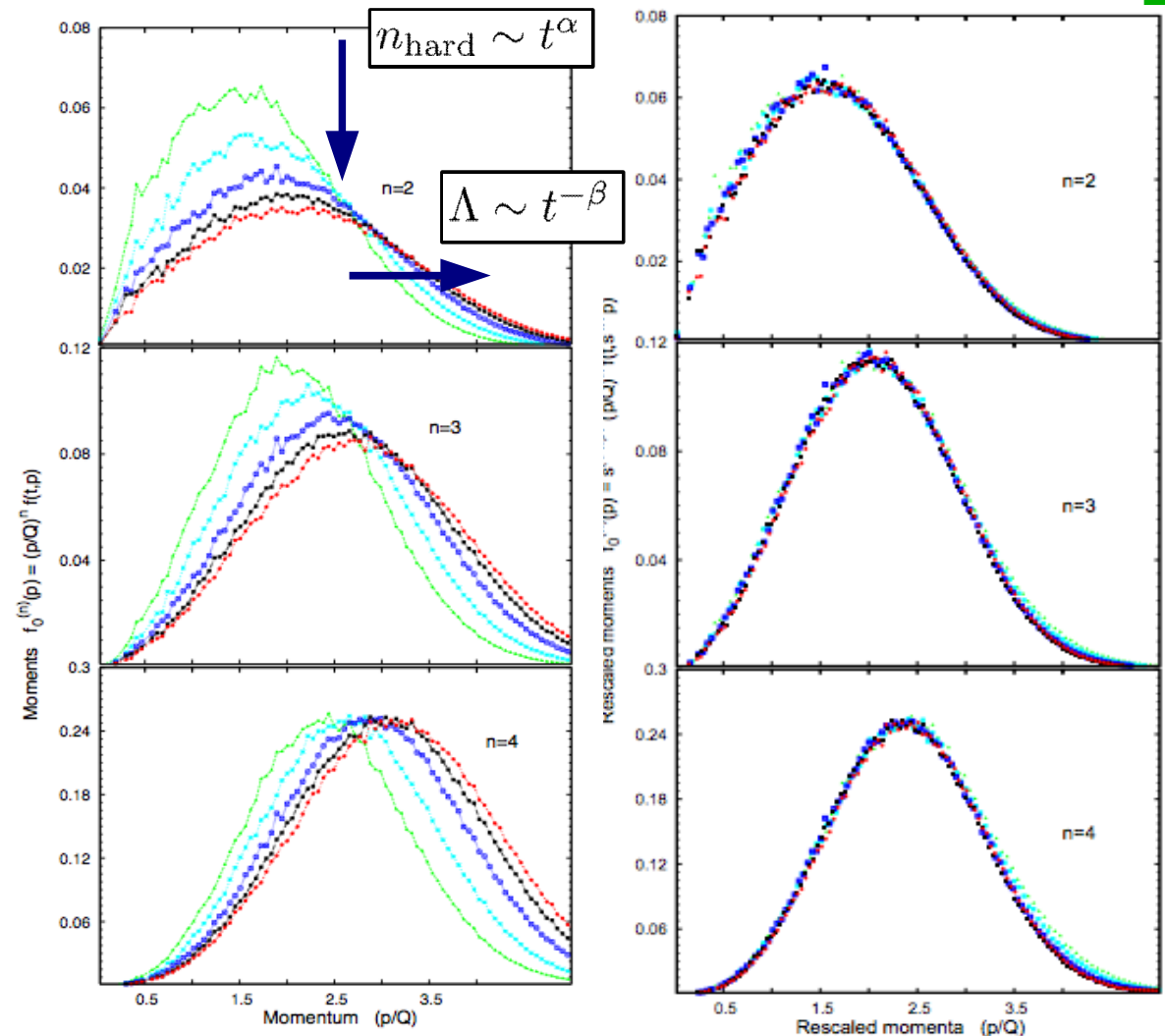
Self-similarity

- Evolution at late times shows a **self-similar** behavior with dynamic scaling exponents

$$\alpha = -4/7$$

$$\beta = -1/7$$

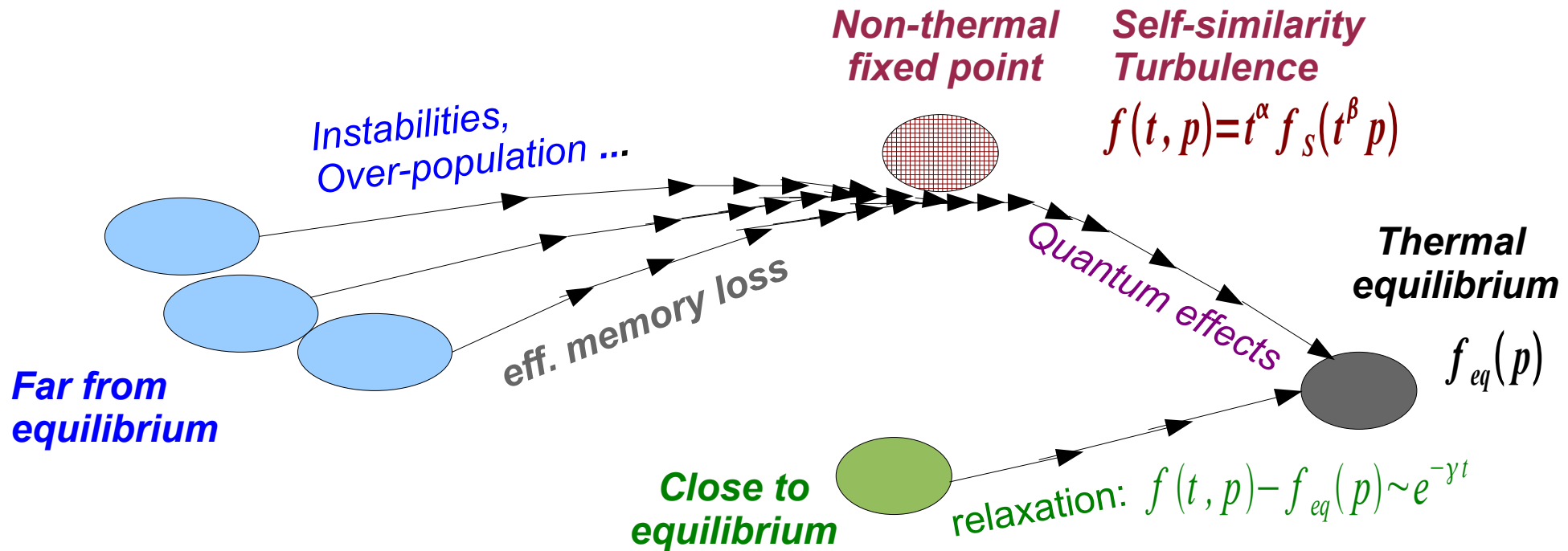
- Consistent with **elastic and inelastic** scattering processes



(SS arXiv:1207.1450 (2012); Kurkela, Moore arXiv:1207.1663 (2012))

Turbulent thermalization

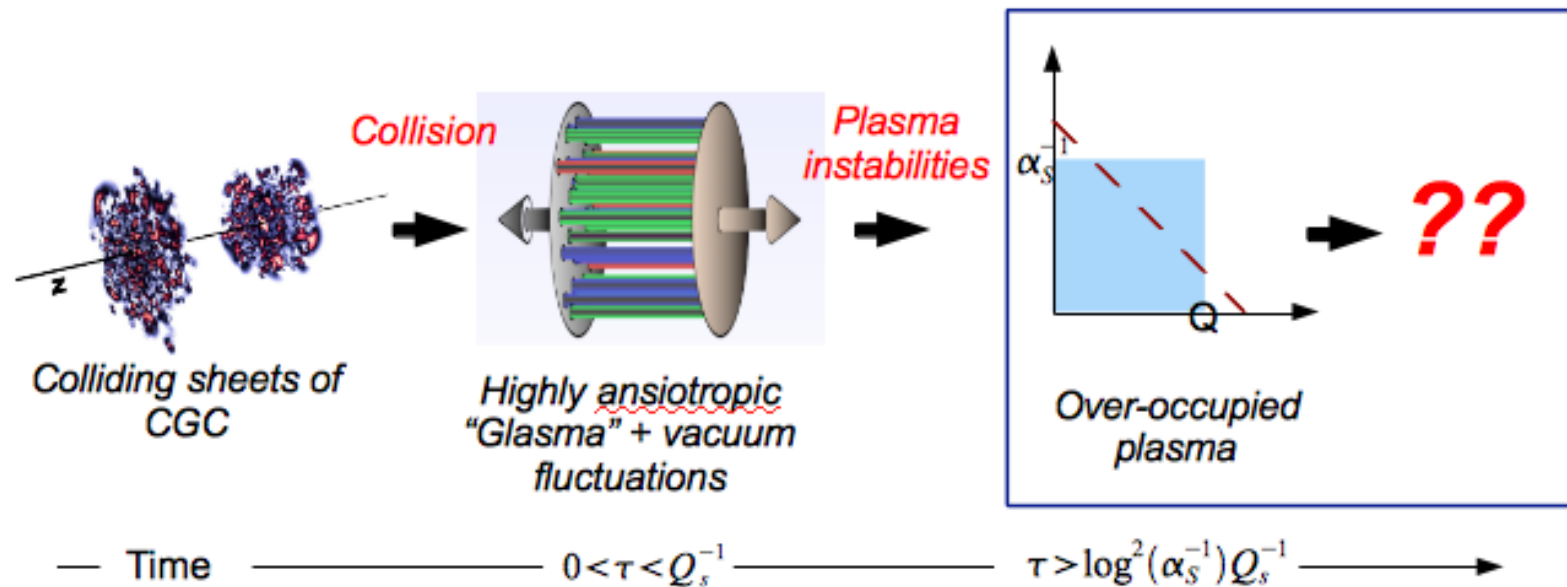
Thermalization for a system far from equilibrium proceeds as a **self-similar evolution** associated to the presence of a **non-thermal fixed point**



*How does this picture apply to non-Abelian gauge theories?
Does it hold for relativistic heavy-ion collisions?*



Heavy-Ion collisions at weak coupling



Strategy:

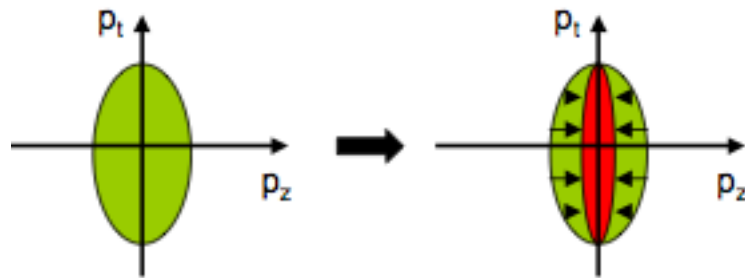
- Choose a large set of initial conditions to mimic the quasi-particle behavior at $\tau_0 \sim \log^2(\alpha_s^{-1}) Q_s^{-1}$ where we start the simulation

$$f(p_T, p_z, \tau_0) = \frac{n_0}{2g^2} \Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

- Simulate for small coupling constants $\alpha_s \sim 10^{-5} \rightarrow Q \tau_0 = 100$ where classical-statistical method is reliable; Extract parametric dependencies and extrapolate to larger couplings.

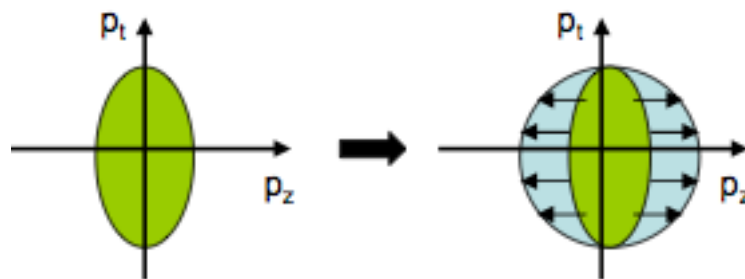
Expanding systems - Thermalization

- The **longitudinal expansion** renders the system **anisotropic** on large time scales. There is a natural **competition** between **interactions** and the longitudinal expansion



Longitudinal Expansion:

- red-shift of longitudinal momenta p
→ increase of anisotropy
- dilution of the system



Interactions:

- isotropize the system

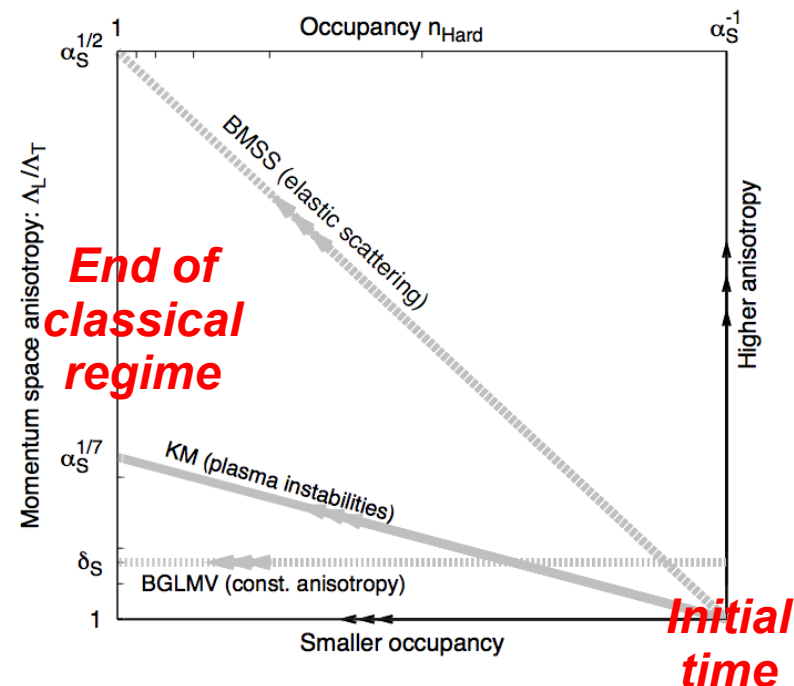
Expanding systems - Thermalization

- Different scenarios of how thermalization proceeds have been proposed in the literature

Baier et al. (*BMSS*),
PLB 502 (2001) 51-58

Kurkela, Moore (*KM*),
JHEP 1111 (2011) 120

Blaizot et al. (*BGLMV*),
Nucl. Phys. A 873 (2012) 68-80

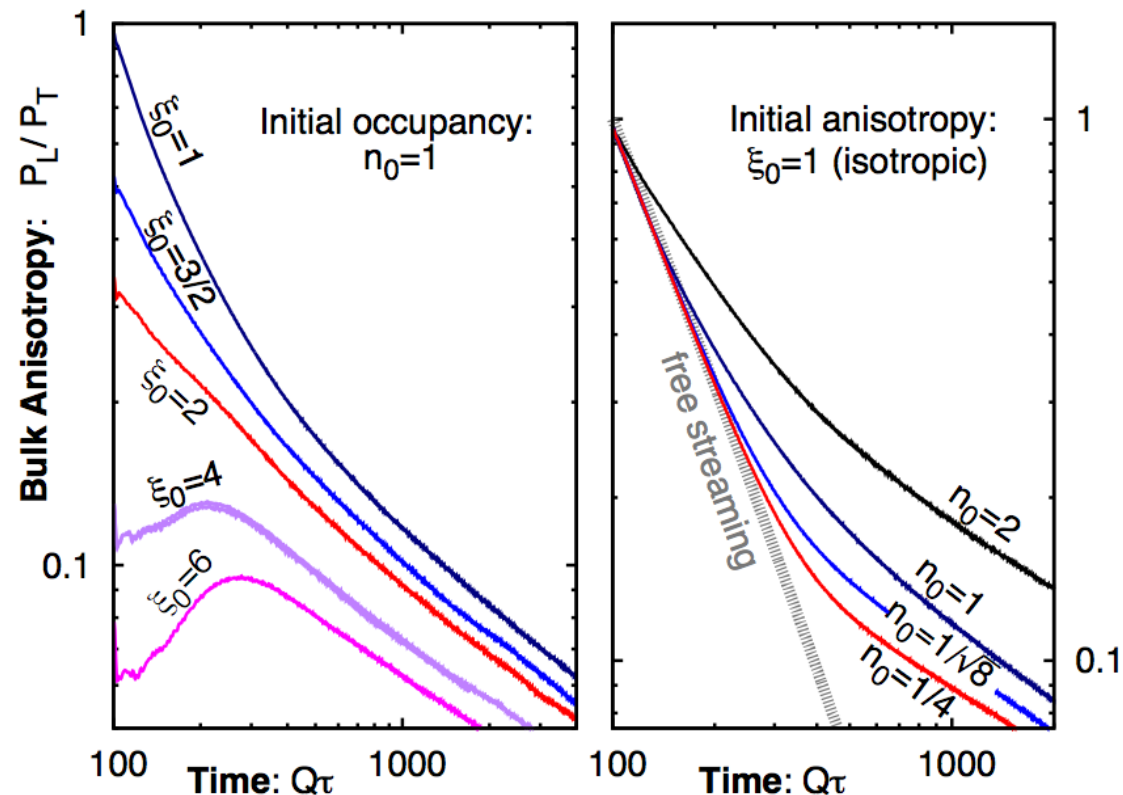


- Difference arises from the treatment of soft (non-perturbative) physics of modes below the Debye scale.

=> Perform non-perturbative classical-statistical lattice simulations up to *256x256x4096* lattices to determine which scenario is realized!

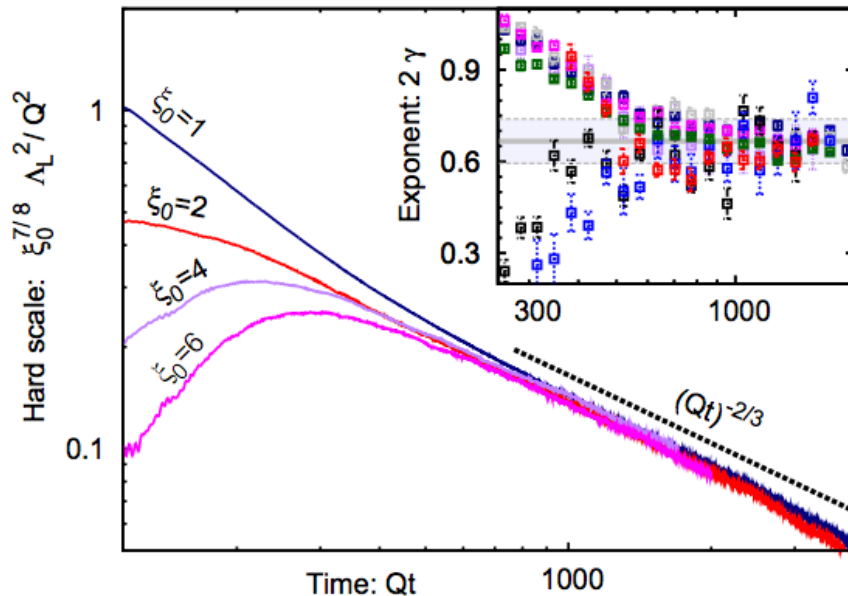
Expanding systems – Bulk anisotropy

- The ***anisotropy*** of the system ***increases*** due to the longitudinal expansion.
- The system remains ***strongly interacting*** throughout the entire evolution.
- At late times, the evolution becomes ***insensitive*** to the details of the ***initial conditions***



(Berges, Boguslavski, SS, Venugopalan arXiv:1303.5650, arXiv:1311.3005)

Expanding systems - Scaling



- The typical **longitudinal momentum** of hard excitations exhibits a **universal scaling** behavior

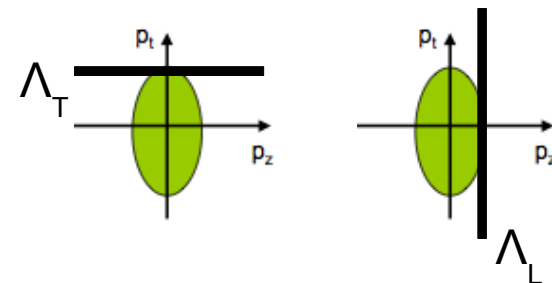
$$\Lambda_L^2 / Q^2 \sim (Qt)^{-2\gamma}$$

$$2\gamma = 0.67 \pm 0.07$$

- The typical **transverse momentum** of hard excitations remains approximately **constant**

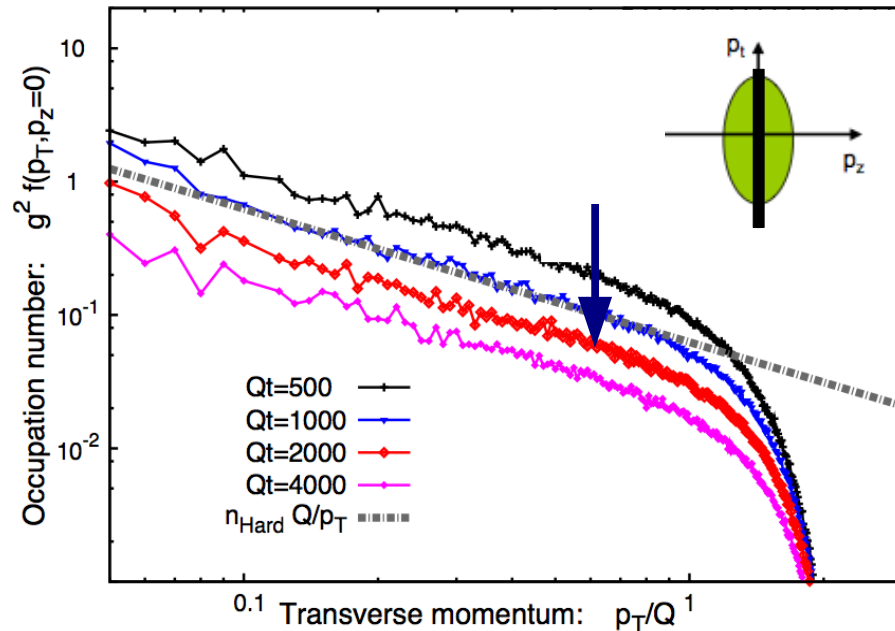
$$\Lambda_T^2 / Q^2 \sim (Qt)^{-2\beta}$$

$$2\beta \simeq 0$$

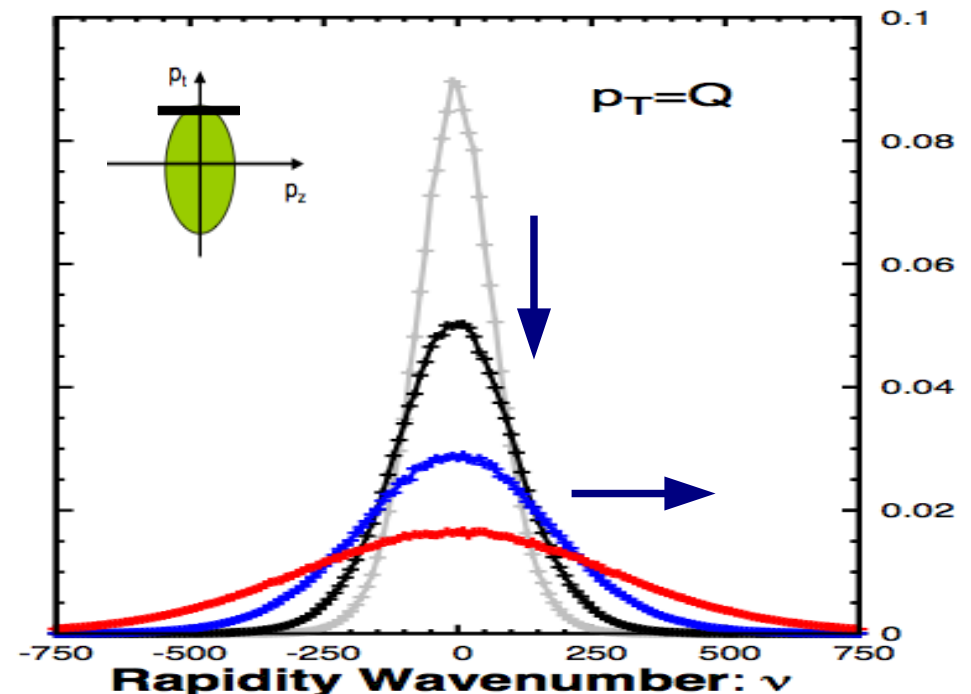


Expanding systems - Spectrum

Transverse spectrum



Longitudinal spectrum

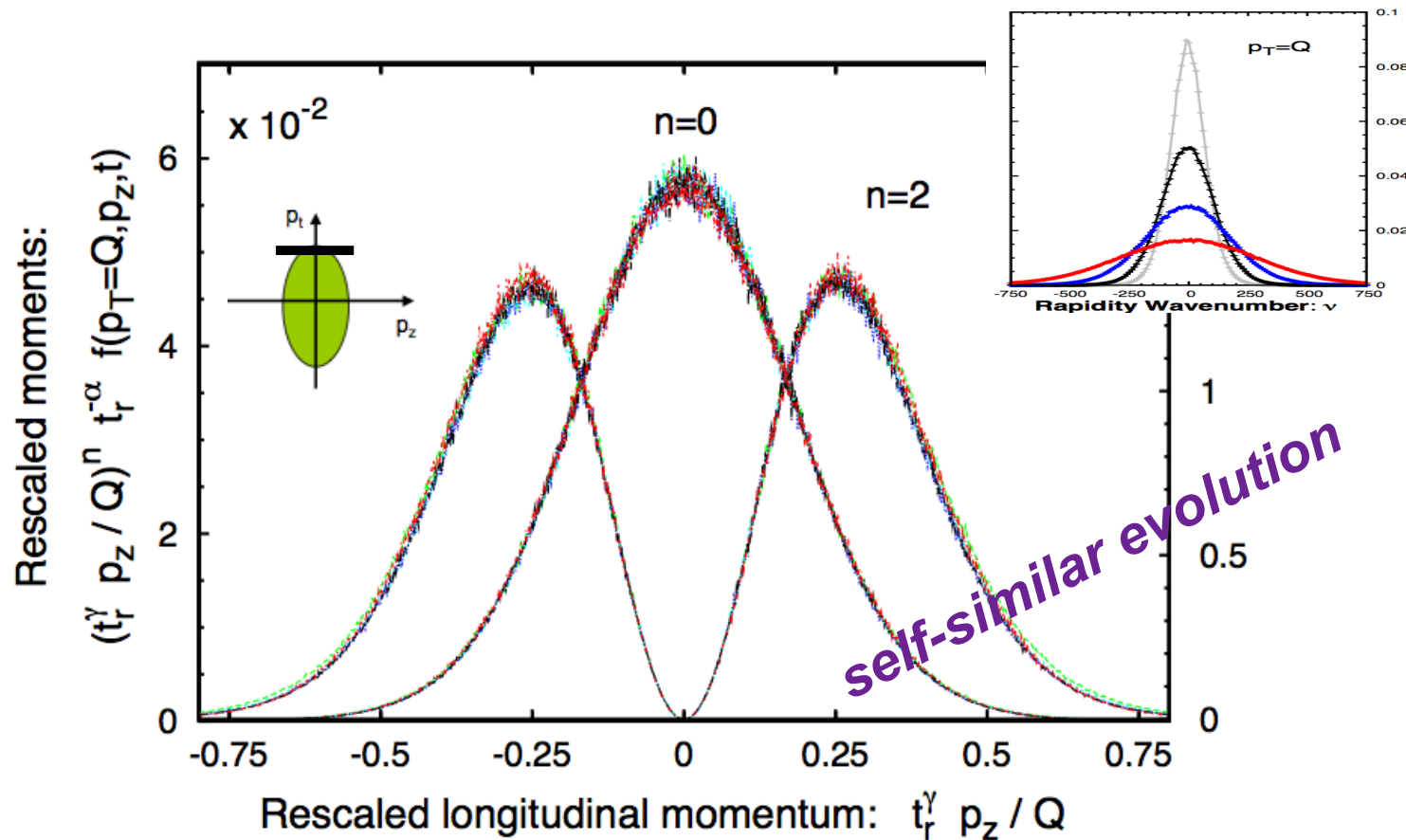


- Thermal like T/p_T spectrum with decreasing amplitude

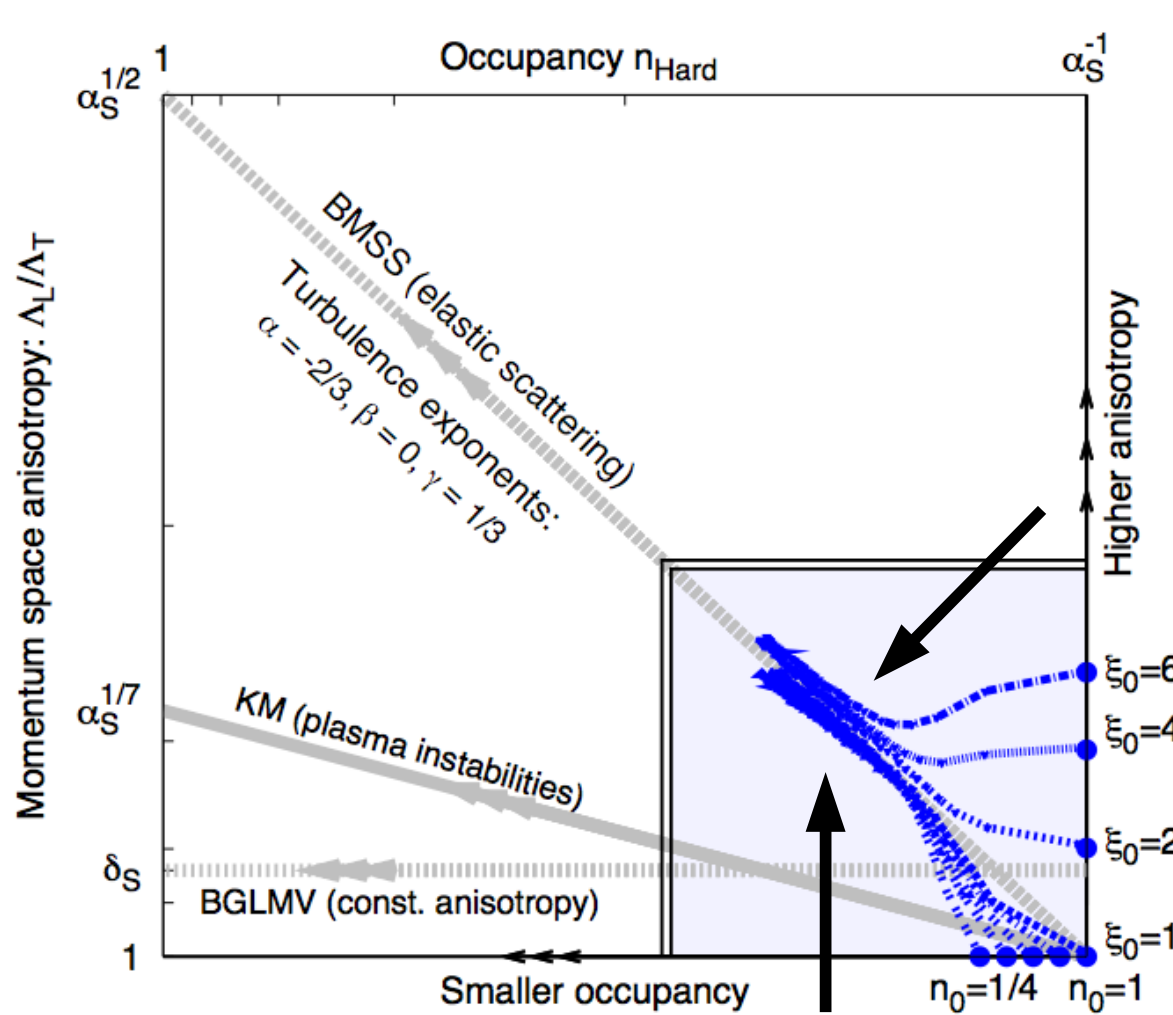
- Continuous momentum broadening – however not strong enough to compensate red-shift

Expanding systems - Self-similarity

- The spectrum of hard excitations shows a ***self-similar evolution*** characteristic of ***wave turbulence***



The attractor solution



- Universal scaling behavior for different initial conditions

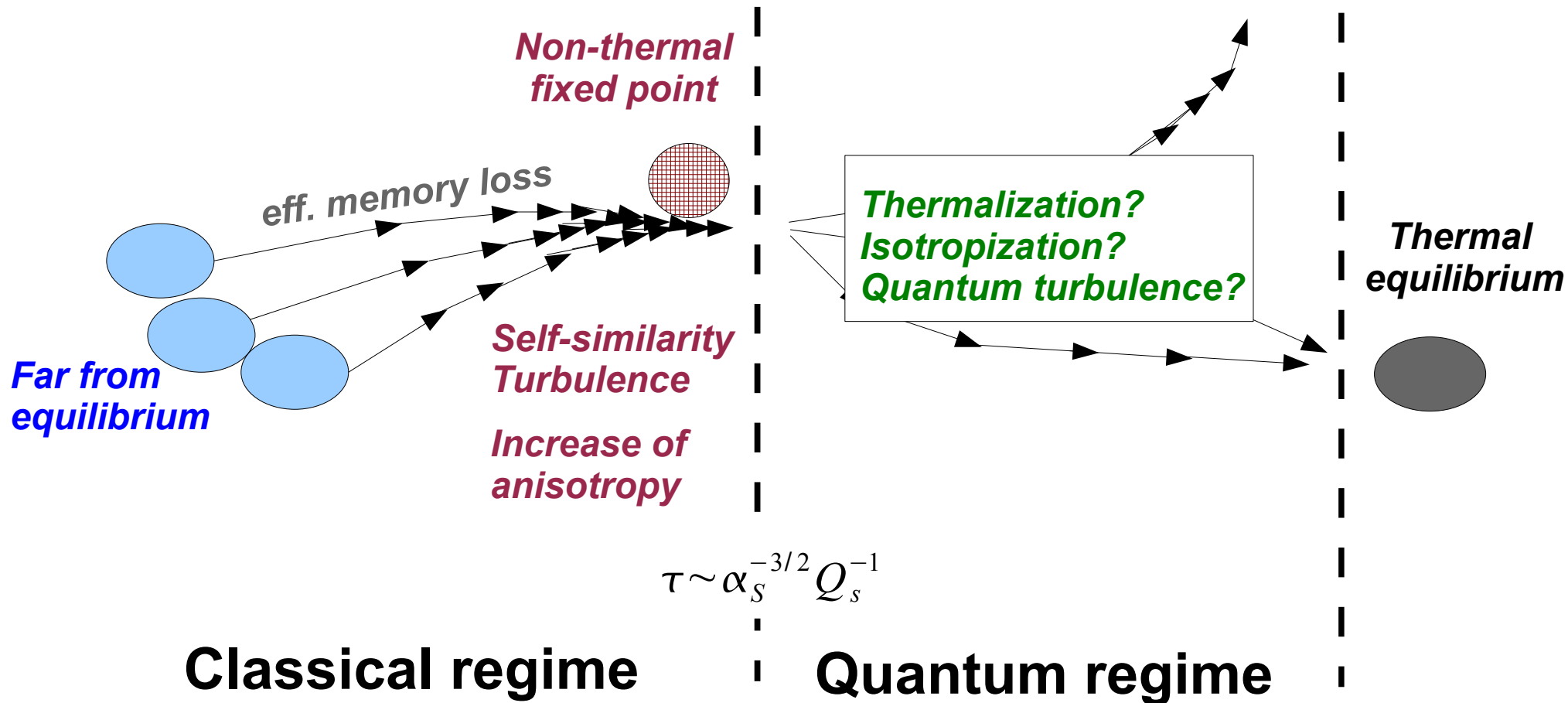
- Self-similar evolution can be characterized by the scaling exponents α, β, γ

$$f(p_T, p_z, \tau) = (Q\tau)^\alpha f_S((Q\tau)^\beta p_T, (Q\tau)^\gamma p_z),$$

- Qualitative agreement with the first stage of the “bottom-up” thermalization scenario (Baier et al. PLB 502 (2001) 51-58)

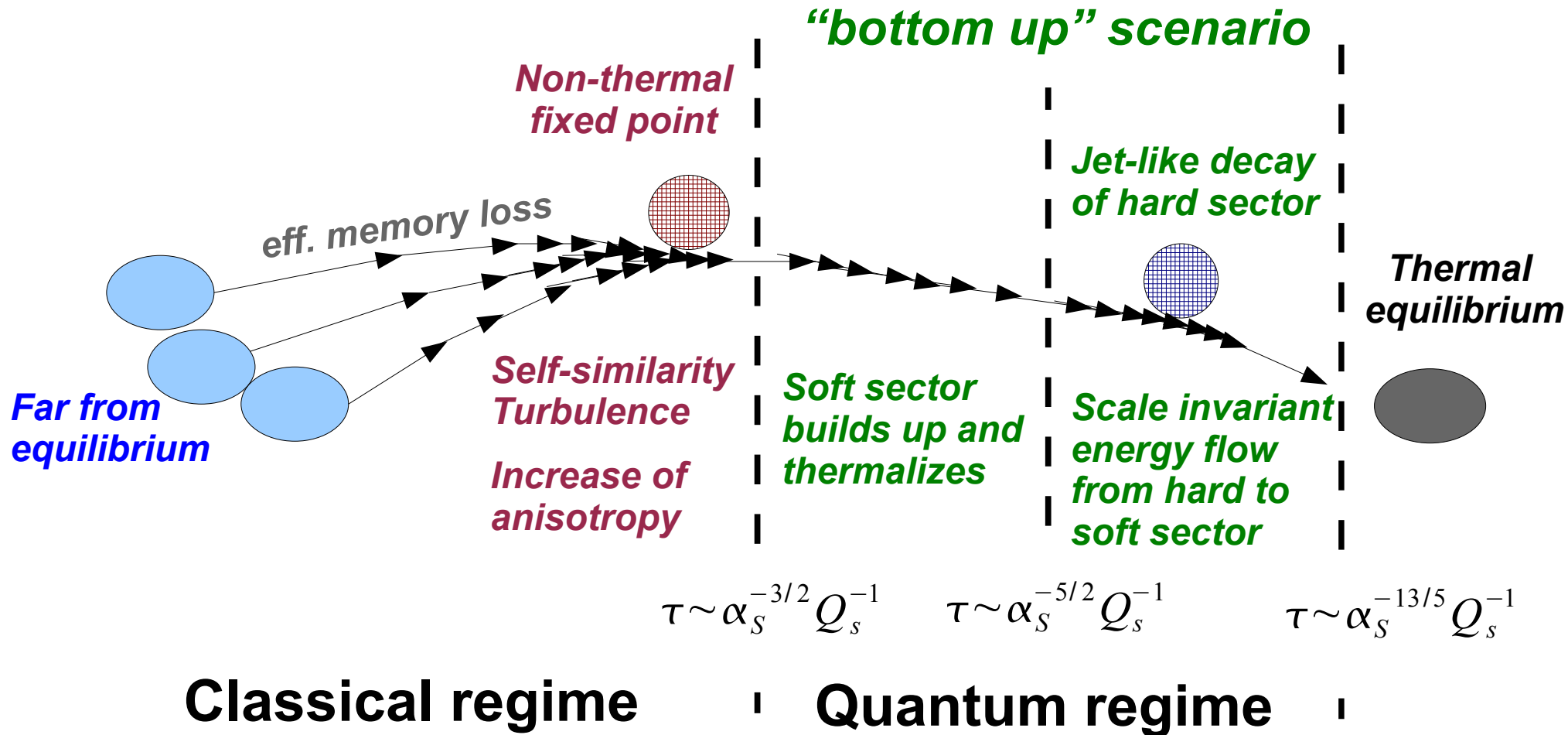
Thermalization of the expanding plasma

The expanding plasma exhibits a ***self-similar evolution***, however at the end of the classical regime the system is ***still far from equilibrium***

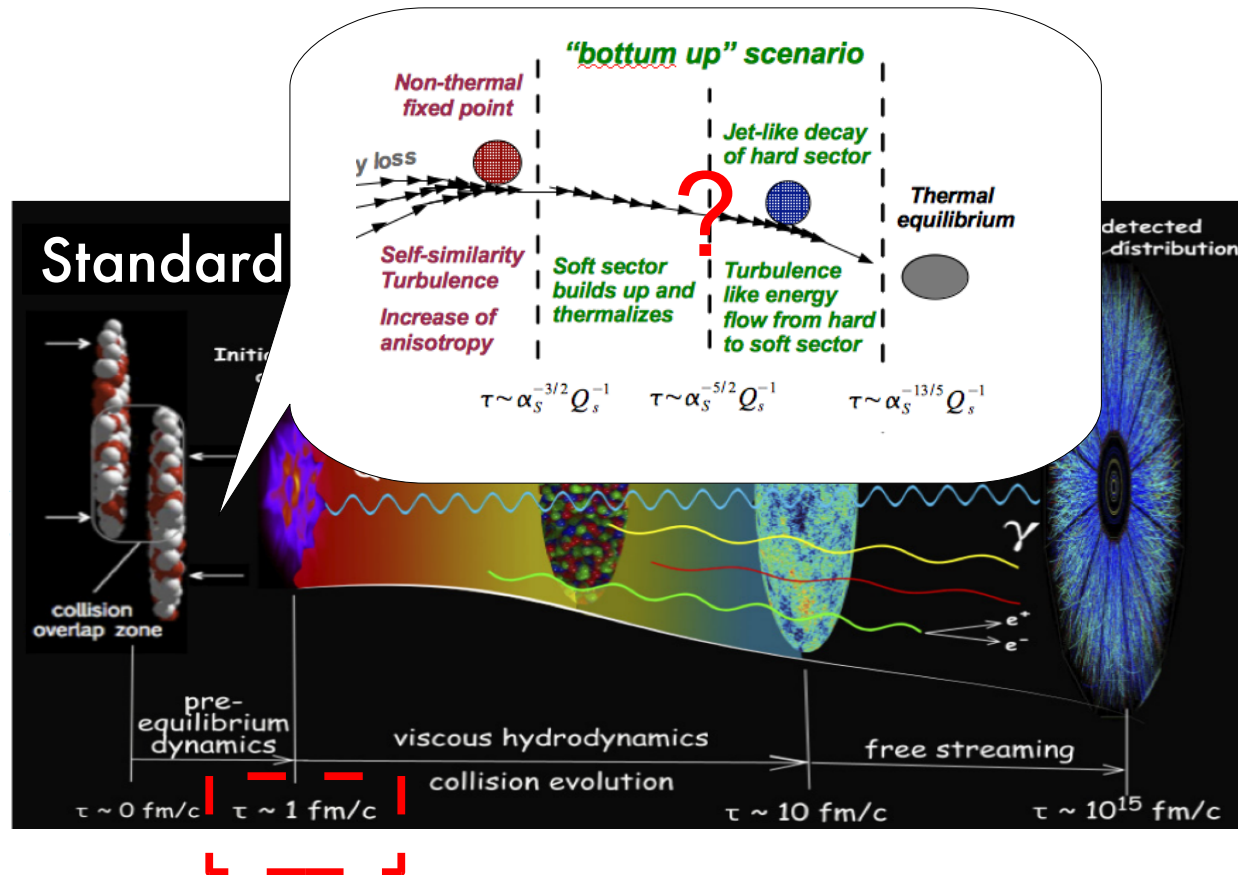


Thermalization of the expanding plasma

Classical statistical simulations no longer applicable in the quantum regime. However kinetic theory predictions provide route to thermal equilibrium



Extrapolation to RHIC/LHC



- Extrapolation to couplings $\alpha \sim 0.3$ give reasonable numbers for the therm. time

$$\tau \sim 2 \text{ fm/c}$$

although pre-factors are undetermined

Conclusion & Outlook

- ***Classical-statistical lattice simulations*** can be used to study the non-equilibrium dynamics from first principles in weak coupling limit.
- The ***thermalization process*** in the classical regime is governed by ***non-thermal fixed points***, where the system exhibits a self-similar evolution characteristic of wave turbulence
- Generic feature of strongly correlated many-body systems across different energy scales (*'big bang', 'little bang', 'ultracold bang'*)

Open questions:

- How is the weak-coupling attractor approached for more realistic initial conditions?
- How exactly is isotropization/thermalization achieved in the quantum regime?
- Can we reliably perform simulations directly at larger values of the coupling?
Is there any change of behavior when the coupling constant increases?

(see also Epelbaum, Gelis PRL 111 (2013) 232301; Berges, Boguslavski, SS, Venugopalan arXiv:1312.5216 (scalars))